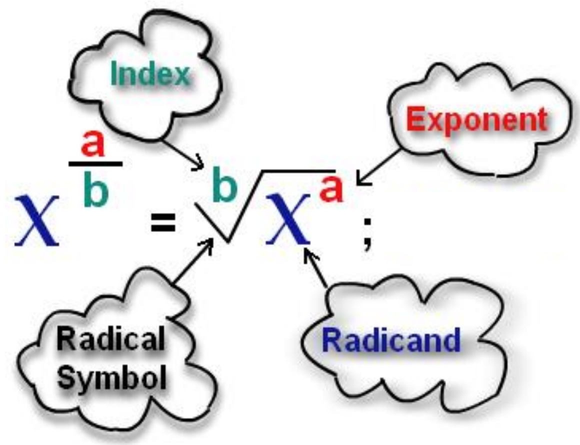
Topic: IM2 – 1.1 (N – 1) Rational Exponents & Radicals Explained	**Standard(s):** N.RN.1	Notes
Essential Question: How can I explain rational exponents and their parts, as well as use notation to show an equivalent expression in radical form?		

Questions / Big Ideas	Key Terms

Rational Exponent ≡ an exponent that is a rational (fractional) number

- If b is an integer greater than 1, then $x^{\frac{a}{b}} = \sqrt[b]{x^a}$



Rational Exponent Rule ≡ $x^{\frac{a}{b}} = \sqrt[b]{x^a} = \left(\sqrt[b]{x}\right)^a$

- The denominator of the fraction is the root.
  - o The denominator of b means b th root (index).
- The numerator of the fraction is the power.
  - o The numerator of a means the a th power (exponent)

It doesn't matter which operation we perform first, the root or the exponent. We can choose!

- Example

  $8^{\frac{2}{3}} = \sqrt[3]{8^2} = \left(\sqrt[3]{8}\right)^2$

  $8^{\frac{2}{3}}$ can be read as 8 to the two-thirds power or the cubed root of 8 squared.

  1. Square First: Square 8 first, then take the cubed root.
     $\sqrt[3]{8^2} = 8^2 = 64$, then $\sqrt[3]{64} = 4$.

  2. Root First: Take the cubed root of 8 first, then square the result.
     $\left(\sqrt[3]{8}\right)^2 = \sqrt[3]{8} = 2$, then $2^2 = 4$.

Questions / Big Ideas

Radical ≡ root symbol

- $\sqrt{x}$ has an invisible index of 2 and an invisible exponent of 1.
    - This is read, “the square root of x.”
    - This can be written as a rational exponent:

$$\sqrt{x} = \sqrt[2]{x^1} = x^{\frac{1}{2}}$$

- $\sqrt[3]{x}$ has an invisible exponent of 1.
    - This is read, “the cube root of x.”
    - This can be written as a rational exponent: $\sqrt[3]{x} = x^{\frac{1}{3}}$

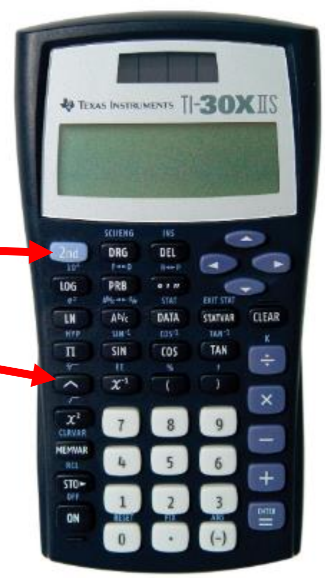
Radicals w/ n^{th} Roots (n: index) ≡ If n is *positive*, then b is the n^{th} root of a.

- Written: $b = \sqrt[n]{a}$

Roots ≡ The root of a number x is another number, which when multiplied by itself a given number of times, equals x

- $b = \sqrt[3]{a}$ means that b, multiplied by itself 3 times, equals a.
- This can be written as $b \cdot b \cdot b = a$ and simplified to $b^3 = a$.
- $\sqrt[3]{a} = b$ is read, “b is the cube root of a,”
  or can be read, “the cube root of a is b.”

- To Calculate the nth Root
    1. Type the number for n (ex. 3)
    2. Press 2nd
    3. Press ^
    4. Type a (ex. 125)
    5. Enter



    - Example (calculated above)

      5 is the cube root of 125, which is written as $\sqrt[3]{125} = 5$;

      so, $5^3 = 125$.

Questions / Big Ideas	The Rational (fractional) Exponent

The Rational (fractional) Exponent

- 1st Focus: Denominator (the nth root)
  - The number of identical factors to expand the original base into.
  - This creates the new base (the factors used in expanded form).

- 2nd Focus: Numerator (exponent)
  - The exponent that one of the repeated factors is raised to, creating a simplified value.

Original Rational Exponent	Expanded Form (base is factored by taking the nth root)	Simplified Value	Explanation of Simplified Value
$16^{\frac{3}{4}}$	Since the 4th root of 16 equals 2, there are 4 repeated factors of 2: $(2\cdot2\cdot2\cdot2)^{\frac{3}{4}}$	$2^3 = \underline{8}$	The simplified value (8) is product of 3 of the 4 repeated factors of the product 16.
$16^{\frac{3}{2}}$	Since the square root of 16 equals 4, there are two repeated factors of 4: $(4\cdot4)^{\frac{3}{2}}$	$4^3 = \underline{64}$	The simplified value (64) is product of 3 of the 2 repeated factors of the product 16.

Equivalent Expressions ≡ expressions with equal values

- Example: $125^{\frac{4}{3}} = (5\cdot5\cdot5)^{\frac{4}{3}} = (5^3)^{\frac{4}{3}} = 5^{\frac{12}{3}} = 5^4 = 625$

Guided Practice

Original = Expanded = Simplified

$243^{\frac{2}{5}} = (\qquad\qquad)^{\frac{2}{5}} =$ _________

Explanation: ____

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Rational Exponent Rule $\equiv x^{\frac{a}{b}} = \sqrt[b]{x^a} = \left(\sqrt[b]{x}\right)^a$

- Rule Applied: $16^{\frac{3}{4}} = \sqrt[4]{16^3} = \left(\sqrt[4]{16}\right)^3$
- Example: Write equivalent expressions for $16^{\frac{3}{4}}$

$$= \left(\sqrt[4]{16}\right)^3 \quad or \quad \sqrt[4]{16^3}$$

$$= \left(\sqrt[4]{2\cdot 2\cdot 2\cdot 2}\right)^3 \quad or \quad \sqrt[4]{(2\cdot 2\cdot 2\cdot 2)^3}$$

$$= \left(\sqrt[4]{2^4}\right)^3 \quad or \quad \sqrt[4]{(2^4)^3}$$

$$= \left(2^{\frac{4}{4}}\right)^3 = (2^1)^3 \quad or \quad \sqrt[4]{2^{12}} = \sqrt[4]{4096}$$

$$= 2^3 = 8 \quad or \quad 2^{\frac{12}{4}} = 2^3 = 8$$

- Questions to Ask Yourself
  1. Can I factor the base into identical integer factors?
  2. Are there more than one set of identical integer factors I can use?
  3. How can I rewrite the base as a power (with an integer exponent)?
  4. Which exponent rules can I apply to this expression?

Guided Practice

Show that both radicals are equivalent using the order of operations.

$$\left(\sqrt[3]{125}\right)^2 \quad = \quad \sqrt[3]{125^2}$$

Summary: ____

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