APPENDIX A: DESIGNING HIGH SCHOOL MATHEMATICS COURSES BASED ON THE COMMON CORE STATE STANDARDS

Integrated Pathway: Mathematics I

The fundamental purpose of Mathematics I is to formalize and extend the mathematics that students learned in the middle grades. The critical areas, organized into units, deepen and extend understanding of linear relationships, in part by contrasting them with exponential phenomena, and in part by applying linear models to data that exhibit a linear trend. Mathematics I uses properties and theorems involving congruent figures to deepen and extend understanding of geometric knowledge from prior grades. The final unit in the course ties together the algebraic and geometric ideas studied. The Mathematical Practice Standards apply throughout each course and, together with the content standards, prescribe that students experience mathematics as a coherent, useful, and logical subject that makes use of their ability to make sense of problem situations.

Critical Area 1: By the end of eighth grade students have had a variety of experiences working with expressions and creating equations. In this first unit, students continue this work by using quantities to model and analyze situations, to interpret expressions, and by creating equations to describe situations.

Critical Area 2: In earlier grades, students define, evaluate, and compare functions, and use them to model relationships between quantities. In this unit, students will learn function notation and develop the concepts of domain and range. They move beyond viewing functions as processes that take inputs and yield outputs and start viewing functions as objects in their own right. They explore many examples of functions, including sequences; they interpret functions given graphically, numerically, symbolically, and verbally, translate between representations, and understand the limitations of various representations. They work with functions given by graphs and tables, keeping in mind that, depending upon the context, these representations are likely to be approximate and incomplete. Their work includes functions that can be described or approximated by formulas as well as those that cannot. When functions describe relationships between quantities arising from a context, students reason with the units in which those quantities are measured. Students build on and informally extend their understanding of integer exponents to consider exponential functions. They compare and contrast linear and exponential functions, distinguishing between additive and multiplicative change. They interpret arithmetic sequences as linear functions and geometric sequences as exponential functions.

Critical Area 3: By the end of eighth grade, students have learned to solve linear equations in one variable and have applied graphical and algebraic methods to analyze and solve systems of linear equations in two variables. This unit builds on these earlier experiences by asking students to analyze and explain the process of solving an equation and to justify the process used in solving a system of equations. Students develop fluency writing, interpreting, and translating between various forms of linear equations and inequalities, and using them to solve problems. They master the solution of linear equations and apply related solution techniques and the laws of exponents to the creation and solution of simple exponential equations. Students explore systems of equations and inequalities, and they find and interpret their solutions. All of this work is grounded on understanding quantities and on relationships between them.

Critical Area 4: This unit builds upon prior students' prior experiences with data, providing students with more formal means of assessing how a model fits data. Students use regression techniques to describe approximately linear relationships between quantities. They use graphical representations and knowledge of the context to make judgments about the appropriateness of linear models. With linear models, they look at residuals to analyze the goodness of fit.

Critical Area 5: In previous grades, students were asked to draw triangles based on given measurements. They also have prior experience with rigid motions: translations, reflections, and rotations and have used these to develop notions about what it means for two objects to be congruent. In this unit, students establish triangle congruence criteria, based on analyses of rigid motions and formal constructions. They solve problems about triangles, quadrilaterals, and other polygons. They apply reasoning to complete geometric constructions and explain why they work.

Critical Area 6: Building on their work with the Pythagorean Theorem in 8th grade to find distances, students use a rectangular coordinate system to verify geometric relationships, including properties of special triangles and quadrilaterals and slopes of parallel and perpendicular lines.

Units	Includes Standard Clusters*	Mathematical Practice Standards
Unit 1 Relationships Between Quantities	 Reason quantitatively and use units to solve problems. Interpret the structure of expressions. Create equations that describe numbers or relationships. 	
Unit 2 Linear and Exponential Relationships	 Represent and solve equations and inequalities graphically. Understand the concept of a function and use function notation. Interpret functions that arise in applications in terms of a context. Analyze functions using different representations. Build a function that models a relationship between two quantities. Build new functions from existing functions. Construct and compare linear, quadratic, and exponential models and solve problems. Interpret expressions for functions in terms of the situation they model. 	Make sense of problems and persevere in solving them. Reason abstractly and quantitatively. Construct viable arguments and critique the reasoning of others. Model with mathematics.
Unit 3 [†] Reasoning with Equations	 Understand solving equations as a process of reasoning and explain the reasoning. Solve equations and inequalities in one variable. Solve systems of equations. 	Use appropriate tools strategically. Attend to precision.
Unit 4 Descriptive Statistics	 Summarize, represent, and interpret data on a single count or measurement variable. Summarize, represent, and interpret data on two categorical and quantitative variables. Interpret linear models. 	Look for and make use of structure. Look for and express regularity in repeated
Unit 5 Congruence, Proof, and Constructions	 Experiment with transformations in the plane. Understand congruence in terms of rigid motions. Make geometric constructions. 	reasoning.
Unit 6 Connecting Algebra and Geometry through Coordinates	Use coordinates to prove simple geometric theorems algebraically.	

^{*}In some cases clusters appear in more than one unit within a course or in more than one course. Instructional notes will indicate how these standards grow over time. In some cases only certain standards within a cluster are included in a unit.

 $^{^{\}dagger}$ Note that solving equations and systems of equations follows a study of functions in this course. To examine equations before functions, this unit could be merged with Unit 1.

Unit 1: Relationships Between Quantities

By the end of eighth grade students have had a variety of experiences working with expressions and creating equations. In this first unit, students continue this work by using quantities to model and analyze situations, to interpret expressions, and by creating equations to describe situations.

Unit 1: Relationships between Quantities		
Clusters with Instructional Notes	Common Core State Standards	
SKILLS TO MAINTAIN Reinforce understanding of the properties of integer exponents. The initial experience with exponential expressions, equations, and functions involves integer exponents and builds on this understanding.		
 Reason quantitatively and use units to solve problems. Working with quantities and the relationships between them provides grounding for work with expressions, equations, and functions. 	N.Q.1 Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays. N.Q.2 Define appropriate quantities for the purpose of descriptive modeling. N.Q.3 Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.	
Interpret the structure of expressions. Limit to linear expressions and to exponential expressions with integer exponents.	 A.SSE.1 Interpret expressions that represent a quantity in terms of its context.* a. Interpret parts of an expression, such as terms, factors, and coefficients. b. Interpret complicated expressions by viewing one or more of their parts as a single entity. For example, interpret P(1+r)ⁿ as the product of P and a factor not depending on P. 	
Create equations that describe numbers or relationships. Limit A.CED.1 and A.CED.2 to linear and exponential equations, and, in the case of exponential equations, limit to situations requiring evaluation of exponential functions at integer inputs. Limit A.CED.3 to linear equations and inequalities. Limit A.CED.4 to formulas with a linear focus.	A.CED.1 Create equations and inequalities in one variable and use them to solve problems. <i>Include equations arising from linear and quadratic functions, and simple rational and exponential functions.</i> A.CED.2 Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales. A.CED.3 Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context. <i>For example, represent inequalities describing nutritional and cost constraints on combinations of different foods.</i> A.CED.4 Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. <i>For example, rearrange Ohm's law V = IR to highlight resistance R.</i>	

Unit 2: Linear and Exponential Relationships

In earlier grades, students define, evaluate, and compare functions, and use them to model relationships between quantities. In this unit, students will learn function notation and develop the concepts of domain and range. They move beyond viewing functions as processes that take inputs and yield outputs and start viewing functions as objects in their own right. They explore many examples of functions, including sequences; they interpret functions given graphically, numerically, symbolically, and verbally, translate between representations, and understand the limitations of various representations. They work with functions given by graphs and tables, keeping in mind that, depending upon the context, these representations are likely to be approximate and incomplete. Their work includes functions that can be described or approximated by formulas as well as those that cannot. When functions describe relationships between quantities arising from a context, students reason with the units in which those quantities are measured. Students build on and informally extend their understanding of integer exponents to consider exponential functions. They compare and contrast linear and exponential functions, distinguishing between additive and multiplicative change. They interpret arithmetic sequences as linear functions and geometric sequences as exponential functions.

Unit 2: Linear and Exponential Relationships		
Clusters with Instructional Notes	Common Core State Standards	
Represent and solve equations and inequalities graphically.	A.REI.10 Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).	
For A.REI.10 focus on linear and exponential equations and be able to adapt and apply that learning to other types of equations in future courses. For A.REI.11, focus on cases where f(x) and g(x) are linear or exponential.	A.REI.11 Explain why the x -coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where $f(x)$ and/or $g(x)$ are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.*	
	A.REI.12 Graph the solutions to a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes.	
Understand the concept of a function and use function notation. Students should experience a variety of types of situations modeled by	F.IF.1 Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then $f(x)$ denotes the output of f corresponding to the input x . The graph of f is the graph of the equation $y = f(x)$.	
functions. Detailed analysis of any particular class of function at this stage is not advised. Students should apply these concepts throughout their	F.IF.2 Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.	
future mathematics courses. Draw examples from linear and exponential functions. In F.IF.3, draw connection to F.BF.2, which requires students to write arithmetic and geometric sequences. Emphasize arithmetic and geometric sequences as examples of linear and exponential functions.	F.IF.3 Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. For example, the Fibonacci sequence is defined recursively by $f(0) = f(1) = 1$, $f(n+1) = f(n) + f(n-1)$ for $n \ge 1$.	

Unit 2: Linear and Exponential Relationships

Clusters with Instructional Notes

Common Core State Standards

• Interpret functions that arise in applications in terms of a context.

For F.IF.4 and 5, focus on linear and exponential functions. For F.IF.6, focus on linear functions and intervals for exponential functions whose domain is a subset of the integers. Mathematics II and III will address other function types.

N.RN.1 and N.RN. 2 will need to be referenced here before discussing exponential models with continuous domains.

Analyze functions using different representations.

For F.IF.7a, 7e, and 9 focus on linear and exponential functions. Include comparisons of two functions presented algebraically. For example, compare the growth of two linear functions, or two exponential functions such as y=3° and y=100·2°.

• Build a function that models a relationship between two quantities.

Limit F.BF.1a, 1b, and 2 to linear and exponential functions. In F.BF.2, connect arithmetic sequences to linear functions and geometric sequences to exponential functions.

Build new functions from existing functions.

Focus on vertical translations of graphs of linear and exponential functions. Relate the vertical translation of a linear function to its y-intercept.

While applying other transformations to a linear graph is appropriate at this level, it may be difficult for students to identify or distinguish between the effects of the other transformations included in this standard.

F.IF.4 For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.*

F.IF.5 Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.*

F.IF.6 Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.*

F.IF.7 Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.*

- a. Graph linear and quadratic functions and show intercepts, maxima, and minima.
- e. Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.

F.IF.9 Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.

F.BF.1 Write a function that describes a relationship between two quantities.*

- a. Determine an explicit expression, a recursive process, or steps for calculation from a context.
- b. Combine standard function types using arithmetic operations. For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model.

F.BF.2 Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.*

F.BF.3 Identify the effect on the graph of replacing f(x) by f(x) + k, k f(x), f(kx), and f(x + k) for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.

Unit 2: Linear and Exponential Relationships		
Clusters with Instructional Notes	Common Core State Standards	
Construct and compare linear, quadratic, and exponential models and solve problems. For F.LE.3, limit to comparisons between exponential and linear models.	F.LE.1 Distinguish between situations that can be modeled with linear functions and with exponential functions.	
	 a. Prove that linear functions grow by equal differences over equal intervals; exponential functions grow by equal factors over equal intervals. 	
	b. Recognize situations in which one quantity changes at a constant rate per unit interval relative to another.	
	 Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another. 	
	F.LE.2 Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).	
	F.LE.3 Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function.	
 Interpret expressions for functions in terms of the situation they model. 	F.LE.5 Interpret the parameters in a linear or exponential function in terms of a context.	
Limit exponential functions to those of the form $f(x) = b^x + k$.		

Unit 3: Reasoning with Equations

By the end of eighth grade, students have learned to solve linear equations in one variable and have applied graphical and algebraic methods to analyze and solve systems of linear equations in two variables. This unit builds on these earlier experiences by asking students to analyze and explain the process of solving an equation and to justify the process used in solving a system of equations. Students develop fluency writing, interpreting, and translating between various forms of linear equations and inequalities, and using them to solve problems. They master the solution of linear equations and apply related solution techniques and the laws of exponents to the creation and solution of simple exponential equations. Students explore systems of equations and inequalities, and they find and interpret their solutions. All of this work is grounded on understanding quantities and on relationships between them.

Unit 3: Reasoning with Equations		
Clusters with Instructional Notes	Common Core State Standards	
Understand solving equations as a process of reasoning and explain the reasoning. Students should focus on and master A.REI.1 for linear equations and be able to extend and apply their reasoning to other types of equations in future courses. Students will solve	A.REI.1 Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.	
exponential equations with logarithms in Mathematics III.		
• Solve equations and inequalities in one variable. Extend earlier work with solving linear equations to solving linear inequalities in one variable and to solving literal equations that are linear in the variable being solved for. Include simple exponential equations that rely only on application of the laws of exponents, such as 5° = 125 or 2° = 1/16.	A.REI.3 Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.	
• Solve systems of equations. Build on student experiences graphing and solving systems of linear equations from middle school to focus on justification of the methods used. Include cases where the two equations describe the same line (yielding infinitely many solutions) and cases where two equations describe parallel lines (yielding no solution); connect to GPE.5, which requires students to prove the slope criteria for parallel lines.	A.REI.5 Prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions. A.REI.6 Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.	

Unit 4: Descriptive Statistics

Experience with descriptive statistics began as early as Grade 6. Students were expected to display numerical data and summarize it using measures of center and variability. By the end of middle school they were creating scatterplots and recognizing linear trends in data. This unit builds upon that prior experience, providing students with more formal means of assessing how a model fits data. Students use regression techniques to describe approximately linear relationships between quantities. They use graphical representations and knowledge of the context to make judgments about the appropriateness of linear models. With linear models, they look at residuals to analyze the goodness of fit.

Unit 4: Descriptive Statistics		
Clusters with Instructional Notes	Common Core State Standards	
Summarize, represent, and interpret data on a single count or measurement	S.ID.1 Represent data with plots on the real number line (dot plots, histograms, and box plots).	
variable. In grades 6 - 8, students describe center and spread in a data	S.ID.2 Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets.	
distribution. Here they choose a summary statistic appropriate to the characteristics of the data distribution, such as the shape of the distribution or the existence of extreme data points.	S.ID.3 Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers).	
 Summarize, represent, and interpret data on two categorical and quantita- tive variables. 	S.ID.5 Summarize categorical data for two categories in two-way frequency tables. Interpret relative frequencies in the context of the data (including joint, marginal, and conditional relative frequencies). Recognize possible associations and trends in the data.	
Students take a more sophisticated look at using a linear function to model the relationship between two	S.ID.6 Represent data on two quantitative variables on a scatter plot, and describe how the variables are related.	
numerical variables. In addition to fitting a line to data, students assess how well the model fits by analyzing residuals.	a. Fit a function to the data; use functions fitted to data to solve problems in the context of the data. Use given functions or choose a function suggested by the context. Emphasize linear and exponential models.	
S.ID.6b should be focused on situations for which linear models are	 b. Informally assess the fit of a function by plotting and analyzing residuals. 	
appropriate.	c. Fit a linear function for scatter plots that suggest a linear association.	
• Interpret linear models.	S.ID.7 Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data.	
Build on students' work with linear relationships in eighth grade and	S.ID.8 Compute (using technology) and interpret the correlation coefficient of a linear fit.	
introduce the correlation coefficient. The focus here is on the computation and interpretation of the correlation coefficient as a measure of how well the data fit the relationship. The important distinction between a statistical relationship and a causeand-effect relationship arises in S.ID.9.	S.ID.9 Distinguish between correlation and causation.	

Unit 5: Congruence, Proof, and Constructions

In previous grades, students were asked to draw triangles based on given measurements. They also have prior experience with rigid motions: translations, reflections, and rotations and have used these to develop notions about what it means for two objects to be congruent. In this unit, students establish triangle congruence criteria, based on analyses of rigid motions and formal constructions. They solve problems about triangles, quadrilaterals, and other polygons. They apply reasoning to complete geometric constructions and explain why they work.

Unit 5: Congruence, Proof, and Constructions		
Clusters and Instructional Notes	Common Core State Standards	
Experiment with transformations in the plane.	G.CO.1 Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc.	
Build on student experience with rigid motions from earlier grades. Point out the basis of rigid motions in geometric concepts, e.g., translations move points a specified distance along a line parallel to a specified line; rotations	G.CO.2 Represent transformations in the plane using, e.g., transparencies and geometry software; describe transformations as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle to those that do not (e.g., translation versus horizontal stretch).	
move objects along a circular arc with a specified center through a specified	G.CO.3 Given a rectangle, parallelogram, trapezoid, or regular polygon, describe the rotations and reflections that carry it onto itself.	
angle.	G.CO.4 Develop definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments.	
	G.CO.5 Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure using, e.g., graph paper, tracing paper, or geometry software. Specify a sequence of transformations that will carry a given figure onto another.	
Understand congruence in terms of rigid motions. Rigid motions are at the foundation of the definition of congruence. Students reason from the basic properties of rigid motions (that they preserve distance and angle), which are assumed without proof. Rigid motions and their assumed properties can be used to establish the usual triangle congruence criteria, which can then be used to prove other theorems.	G.CO.6 Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure; given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent. G.CO.7 Use the definition of congruence in terms of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent. G.CO.8 Explain how the criteria for triangle congruence (ASA, SAS, and SSS) follow from the definition of congruence in terms of rigid motions.	
Make geometric constructions. Build on prior student experience with simple constructions. Emphasize the ability to formalize and defend how these constructions result in the desired objects. Some of these constructions are closely related to previous standards and can be introduced in conjunction with them.	G.CO.12 Make formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc.). Copying a segment; copying an angle; bisecting a segment; bisecting an angle; constructing perpendicular lines, including the perpendicular bisector of a line segment; and constructing a line parallel to a given line through a point not on the line. G.CO.13 Construct an equilateral triangle, a square, and a regular hexagon inscribed in a circle.	

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Unit 6: Connecting Algebra and Geometry Through Coordinates

Building on their work with the Pythagorean Theorem in 8th grade to find distances, students use a rectangular coordinate system to verify geometric relationships, including properties of special triangles and quadrilaterals and slopes of parallel and perpendicular lines.

Unit 6: Connecting	Algebra and Geometry	Through Coordinates
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Clusters and Instructional Notes

• Use coordinates to prove simple geometric theorems algebraically.

This unit has a close connection with the next unit. For example, a curriculum might merge G.GPE.1 and the Unit 5 treatment of G.GPE.4 with the standards in this unit. Reasoning with triangles in this unit is limited to right triangles; e.g., derive the equation for a line through two points using similar right triangles.

Relate work on parallel lines in G.GPE.5 to work on A.REI.5 in Mathematics I involving systems of equations having no solution or infinitely many solutions.

G.GPE.7 provides practice with the distance formula and its connection with the Pythagorean theorem.

Common Core State Standards

G.GPE.4 Use coordinates to prove simple geometric theorems algebraically. For example, prove or disprove that a figure defined by four given points in the coordinate plane is a rectangle; prove or disprove that the point $(1, \sqrt{3})$ lies on the circle centered at the origin and containing the point (0, 2).

G.GPE.5 Prove the slope criteria for parallel and perpendicular lines; use them to solve geometric problems (e.g., find the equation of a line parallel or perpendicular to a given line that passes through a given point).

G.GPE.7 Use coordinates to compute perimeters of polygons and areas of triangles and rectangles, e.g., using the distance formula.*